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SCATTERING OF A CODE-MODULATED RADIO SIGNAL AND ASSOCIATED MULT--ETC(U)
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SCATTERING OF A CODE-MODULATED RADIO SIGNAL
AND ASSOCIATED MULTIPATH RANGE ERRORS

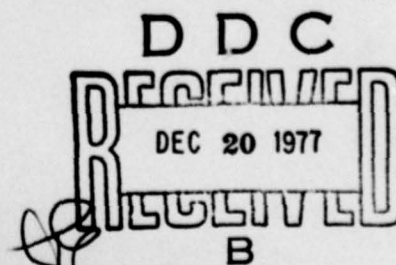
by
Dr. Eugene A. Margerum
and
Dr. Frederick W. Rohde

January 1977

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Multipath	Gold Code									
Scattering	Code Modulation									
Radar	Carrier Cancellation									
Positioning	Ranging									
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) <p>→ To account for range errors obtained when a code-modulated radio signal is used to make range measurements in a scattering environment, a mechanism termed "carrier cancellation" is proposed. This allows for destructive interference between several nearly direct path signals, leaving the possibility for a weaker signal that has traveled over a longer path to predominate. A mathematical formulation of the phenomenon is given, and the equations derived are used for a computer simulation of the effects. The results of the simulation confirm that the proposed mechanism represents a viable explanation of the observed range errors. ←</p>										

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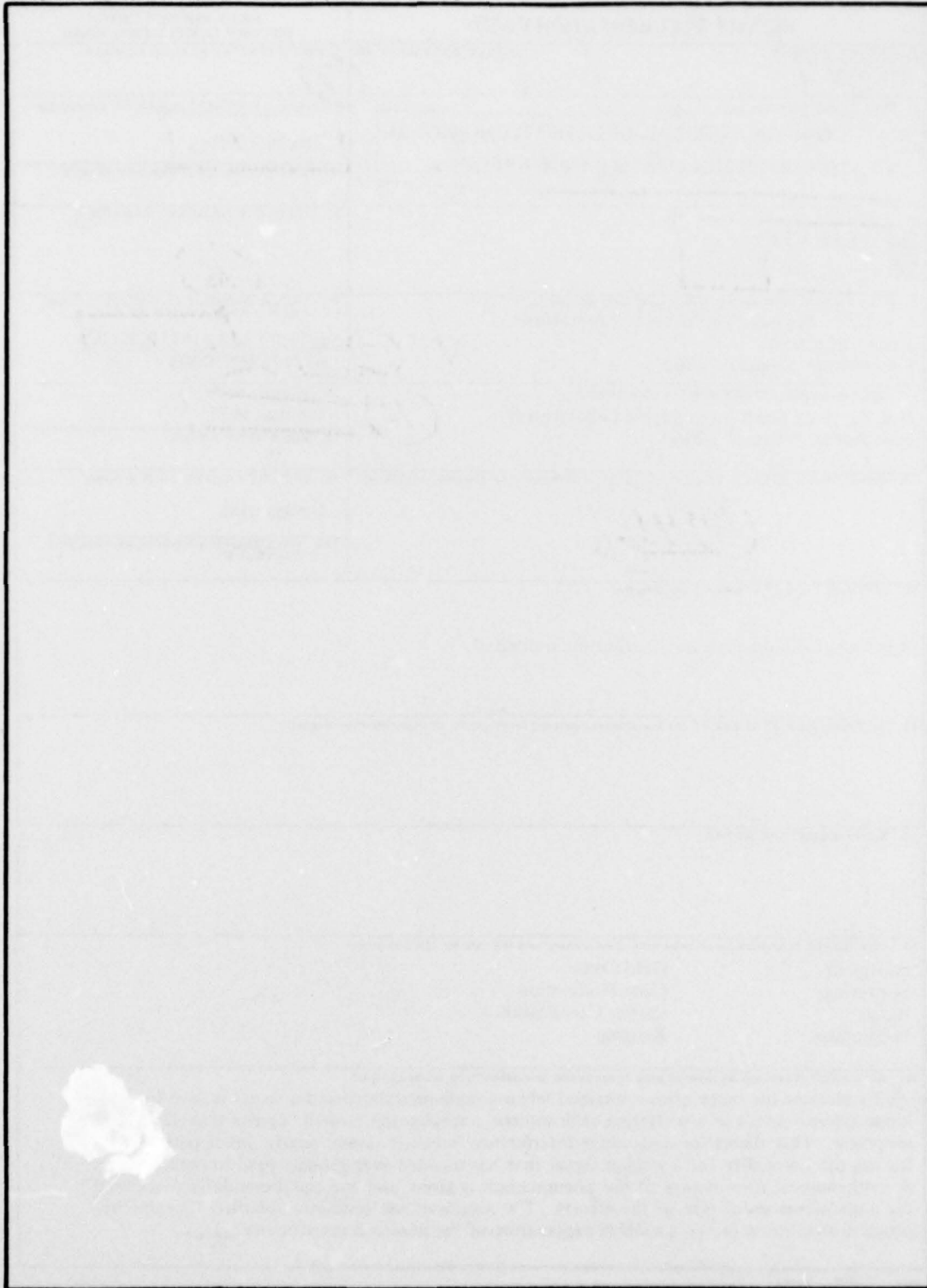
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PREFACE

The work reported here was undertaken in connection with a position determining system that uses the ranges from a receiver on the ground to several satellites having known positions to determine coordinates of the ground location. These ranges are proportional to the delay time between transmission by the satellite and reception on the ground. The delays are determined by correlation with a signal generated on the ground that is a replica of the satellite signal. Since it is desirable to use the system in the presence of vegetation and other potential scatterers, it is important to know the characteristics and extent of errors that can be introduced by multipath signals caused by such scatterers. Field measurements had been performed in the presence of vegetation and the results indicated that anomalous range measurements sometimes occur. The present study is intended to provide a plausible explanation for anomalous range measurements obtained in such a scattering environment.

The work was performed in the Center for Theoretical and Applied Physical Sciences in the Research Institute of the U.S. Army Engineer Topographic Laboratories, under Project 4A161102B52C, Task S3, Work Unit 1752CS30003.

The authors are especially grateful to Mr. Jack Jacobson and his co-workers for making available the results of the field experiments in a reduced and usable form.

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SCATTERING OF A CODE-MODULATED RADIO SIGNAL AND ASSOCIATED MULTIPATH RANGE ERRORS

INTRODUCTION

Objective. In making range measurements using a code-modulated radio signal in the presence of scatterers, sometimes large range measurement errors were obtained. In all cases, the range measurements obtained were larger than the actual range. Often, a proper measurement of the range could be restored if the receiving antenna was moved less than three-tenths of a meter (about 1 foot). Figures 1 through 3 show examples of actual measurements. The purpose of this study was to develop a mathematical model that would account for the anomalous range measurements and would also provide for computations leading to a numerical simulation of these measurements. The computations were to be performed for some representative cases to demonstrate the plausibility of the model and to give some insight into the mechanisms and relative significance of the various parameters involved.

The basic scattering phenomena upon which the characteristics of the received signal ultimately depend were not investigated because such an undertaking would have exceeded the scope of this study. It is sufficient to know that diffraction and scattering of radio waves in foliage will generate interference patterns with spatial distributions of electromagnetic energy that show considerable local variations. For that reason, the effects of scattering were included only in the form of delayed multipath signals with somewhat arbitrarily assigned delay times and amplitudes. Also, no attempt was made to model the receiver in detail, but it was considered to possess idealized characteristics of receivers in general.

The Experimental Configuration to be Simulated. The distance between two points is measured by determining the travel time for a signal between a transmitter and receiver as shown in figure 4. A biphase code-modulated signal sent by the transmitter is received and correlated with an identical signal produced at the receiver. By using an accurately synchronized system of clocks, the time delay is obtained and hence, the corresponding range or distance from the transmitter to the receiver.

When such a system is used in an environment containing scatterers such as trees and foliage, several signals transmitted over different paths can possibly be received (figure 5). In such surroundings, it is important that the wave propagated over the most direct path transports sufficient energy that the receiver may lock onto this component; otherwise, the receiver may lock onto another component and give a faulty range. One mechanism that can cause this to happen is destructive interference

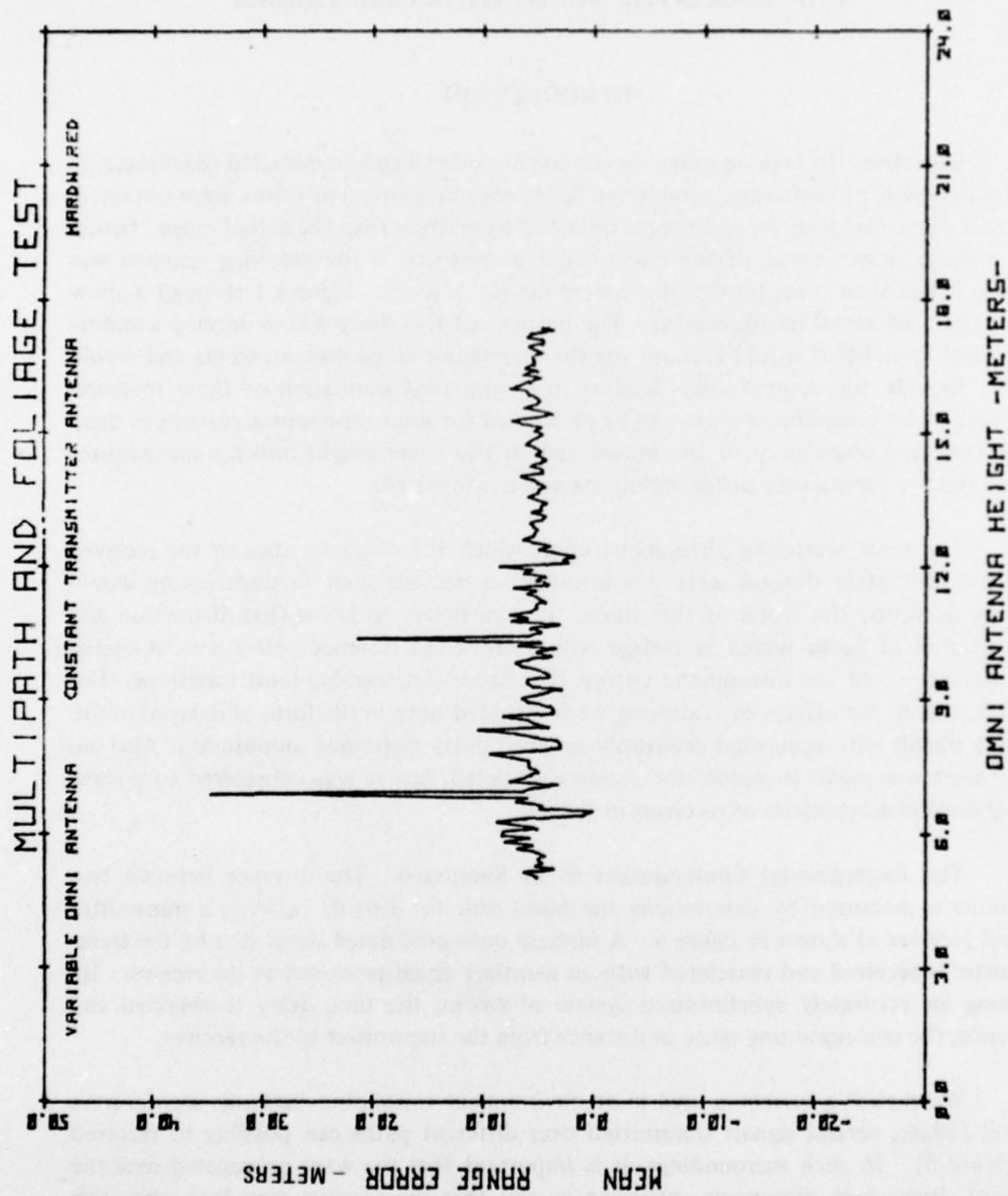


Figure 1. One Example of Range Errors Caused by Carrier Cancellation.

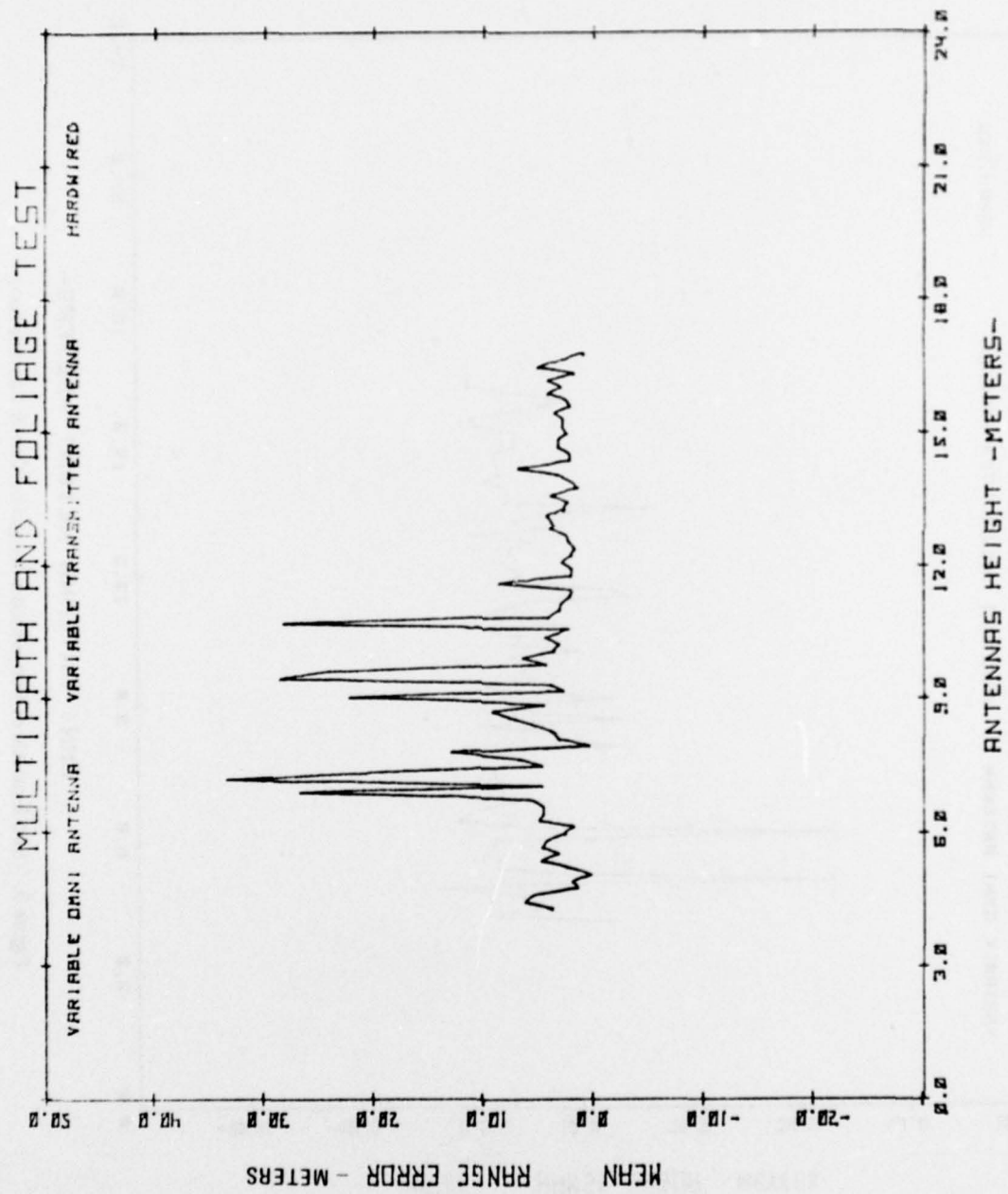


Figure 2. Second Example of Range Errors Caused by Carrier Cancellation.

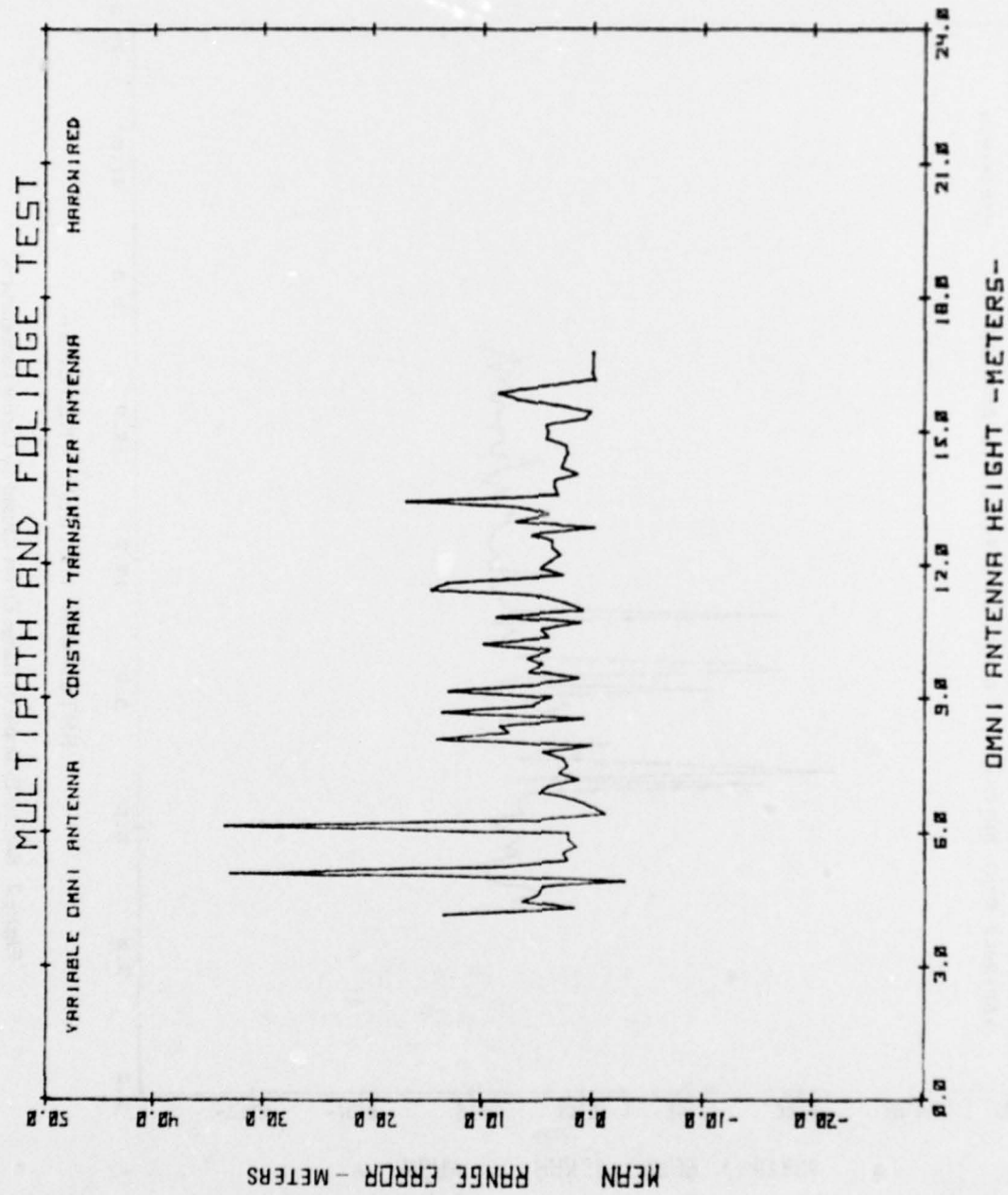


Figure 3. Third Example of Range Errors Caused by Carrier Cancellation.

At each discontinuity of the modulated signal, the phase of the carrier is shifted by 180° (figure 7).

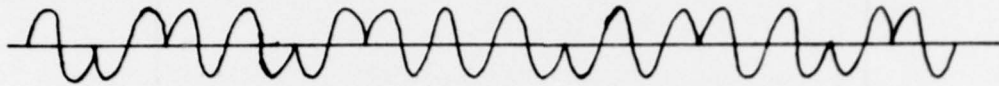


Figure 7. Biphase Modulated Carrier.

The basic lengths, periods, and frequencies describing the carrier, pulses, and code, together with the other parameters used, are given in Table 1.

Table 1.

	Length	Period	Frequency
Carrier	$1.903429 \times 10^{-1} \text{ m}$ ($\sim 20 \text{ cm}$) ($\sim 8 \text{ in.}$)	$6.349841 \times 10^{-10} \text{ sec}$ ($\sim 6 \text{ nanoseconds}$)	$1.575 \times 10^9 \text{ Hz}$ (1.575 GHz)
Pulse	$2.997925 \times 10^1 \text{ m}$ ($\sim 30 \text{ m}$) ($\sim 33 \text{ yd}$)	10^{-7} sec ($\sim 0.1 \text{ microsecond}$) ($\sim 100 \text{ nanoseconds}$)	$1.0 \times 10^7 \text{ bit/sec}$ (10 megabits/sec)
String	$3.0669 \times 10^4 \text{ m}$ ($\sim 31 \text{ km}$) ($\sim 19 \text{ mi}$)	$1.023 \times 10^{-4} \text{ sec}$ ($\sim 0.1 \text{ milliseconds}$)	$9.77517 \times 10^3 \text{ sec}^{-1}$ ($\sim 10,000/\text{sec}$)

$$\text{Signal Velocity} = C = 2.997925 \times 10^8 \text{ m s}^{-1}$$

$$\text{Pulse Length} \cong 157.5 \text{ Wavelengths}$$

$$\text{Code Length} = 1,023 \text{ Pulses}$$

The various parameters given are important in determining the ranges of path length difference over which the different mechanisms to be described become operative.

As an example, consider the case when the strongest and most direct component of the incident signal hits an obstacle, such as a trunk or branch of a tree. The obstacle may diffract the signal into a diffraction pattern having a characteristic spatial intensity distribution. At certain locations of the pattern, the intensity may be very small. A receiver that is locked to this signal may lose it if the receiver antenna is moved to a low intensity point of the diffraction pattern. If there is another sufficiently strong

but delayed component, the receiver may acquire it at this point and the resulting range measurement would produce a faulty range.

The intensity variations in the diffraction pattern can be explained by the processes of constructive and destructive interference. This type of interference is observed when coherent signals or waves are superimposed. However, incoherent signals or waves do not produce observable interference patterns. Strictly speaking, destructive interference doesn't exist for pseudo-noise biphase-modulated signals because the phase of the carrier changes almost randomly by 180° in accordance with the modulation. If the path difference of two components is larger than the modulation pulse length, the interference at a given point changes on the average with each clock pulse. Because of the very high clock rate, no destructive interference can be observed. For path differences less than the modulation pulse length, destructive interference may be observed. If ℓ is the path difference and L the pulse length and $0 \leq \ell \leq L$, the observable portion of the interference is $L - \ell/L$, and the nonobservable portion is ℓ/L .

However, if two signals were present, but separated in path length by about a pulse length (30 meters) or more, the tendency toward carrier cancellation would depend on the autocorrelation function for the code. The actual codes used are Gold Codes possessing pseudorandom characteristics; therefore, beyond a relative path difference of about a pulse length (30 m), the signals tend to interfere constructively about as often as destructively, and the mechanism of carrier cancellation is inoperative. In the past, the combination of (1) normal operation in a nonscattering environment, and (2) over-emphasis of the dependence of the signal correlation on the code correlation properties led to the neglect of possible carrier interference effects when operating in a scattering or multipath environment. Consequently, the importance of the phenomenon of carrier cancellation was underrated.

The actual codes used in the simulations were shorter than the codes normally used in the field, but they possessed similar characteristics. The code generation system that was simulated consisted of a pair of shift registers, connected as shown in figure 8. Each shift register consists of 10 consecutive binary storage elements called stages and a feedback network. The binary values (e.g. two discrete voltages) that can be stored in each stage are for the purpose of this treatment defined as value zero (0) and value one (+1). All stages of the two shift registers in figure 6 show the value (+1). The feedback network feeds the values of some designated stages into a modulo two adder. Depending on the input values of the adder, the output of the adder will assume the values 0 or +1. The output of the adder is applied to the input of the shift register. The outputs of both shift registers are fed into a modulo two adder that generates the Gold Code modulation $s(t)$. If the values at the output of both shift registers are equal, a value of (+1) is assigned to the code; if they are unequal, a value of (-1) is

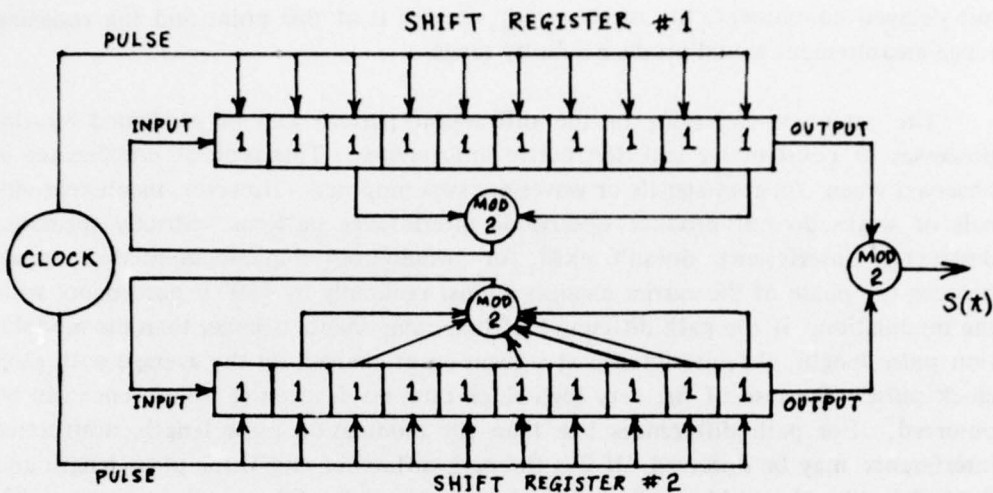


Figure 8. Gold Code Generation.

assigned to the code. The shift registers are operated by applying a clock pulse to each stage. The application of a clock pulse causes the value of any stage to be shifted to the next stage on the right and the output of the feedback network to be shifted into the first stage of the shift register. To initialize the registers, each element of the registers was set to +1. Provision was made in the computer program for representing different feedback connections (only certain ones would be capable of generating a maximum length code). Also one of the registers could be advanced independently of the other to change the code and to determine the effects of such changes. An example of one of the Gold Codes actually used is given in figure 9. The autocorrelation function for this code is given in figure 10. Its principal correlation value is 1,023 and is determined by the integration range that, in this case, is equal to the code length. The other values assumed by the correlation function are 63, -65, and -1. It would seem desirable to use a code that has correlation values of -1 as the only value occurring near the principal correlation value of 1,023. Some simulations have shown that if the code is improved in this respect by advancing one shift register relative to the other, the ability to correlate can be improved.

MATHEMATICAL ANALYSIS

Correlation of the Received Multicomponent Signal with a Reference Signal.

To perform the simulations, it was necessary to derive the computational equations and to formulate a method to reduce the computations to a tractable size. Also, the resulting equations should lend themselves to a convenient interpretation of the problem. The signal was assumed to be repeated periodically without pause. The equations were presented in the time domain rather than in the spatial domain.

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The signal produced by the transmitter is given by

$$S(t) = A s(t) \cos \omega t \quad (1)$$

where A is the amplitude, ω is the carrier frequency, t is the time and $s(t)$ is the modulation representing the code ($s(t) = \pm 1$). The signal at the receiver $S_r(t)$ is the superposition of a number M of component waves $S_j(t)$; each being a replica of the transmitted signal and having an amplitude A_j and a time delay τ_j .

$$S_r(t) = \sum_{j=1}^M S_j(t) \quad (2)$$

$$S_r(t) = \sum_{j=1}^M A_j s(t - \tau_j) \cos \omega(t - \tau_j) \quad (3)$$

The delays τ_i must account for (1) increased travel distance, (2) altered velocity of propagation, and (3) phase changes due to scattering. For purposes of computation, they are expressed in terms of equivalent ranges (optical paths) r_i ,

$$r_i = c \tau_i \quad (4)$$

where c is the velocity of propagation in a vacuum.

At the receiver, the modulated signal is separated from the carrier. This is represented mathematically by multiplying the incoming signal $S_r(t)$ with an internally generated signal $S_R(t) = A_o \cos \omega(t - \tau_o)$. The demodulated signal $S_m(t)$ is

$$S_m(t) = S_r(t) S_R(t) = \sum_{j=1}^M A_o A_j s(t - \tau_j) \cos \omega(t - \tau_j) \cos \omega(t - \tau_o) \quad (5)$$

$$S_m(t) = \sum_{j=1}^M A_o A_j s(t - \tau_j) \left\{ \frac{1}{2} \cos \omega(2t - \tau_j - \tau_o) + \frac{1}{2} \cos \omega(\tau_j - \tau_o) \right\} \quad (6)$$

In practice, demodulation may be accomplished by a somewhat different procedure, but the result will correspond ideally with the mathematical results given here. The term in (6) with the factor $\frac{1}{2} \cos \omega(2t - \tau_j - \tau_o) = \frac{1}{2} \cos (2\omega t - \phi)$ represents the high frequency component with the angular frequency 2ω . This component can be filtered out by a low pass filter. The other component of (6) represents the detected signal,

$$S_d = \sum_{j=1}^M \frac{A_o A_j}{2} \cos \omega(\tau_j - \tau_o) s(t - \tau_j) \quad (7)$$

or

$$S_d = \sum_{j=1}^M B_j s(t - \tau_j) \quad (8)$$

with

$$B_j = \frac{A_o A_j}{2} \cos \omega(\tau_j - \tau_o)$$

Equations (3) and (7) are suitable to demonstrate that carrier cancellation is a property of the scattered signal and cannot be influenced by receiver manipulations. Consider the case that the signal $S_r(t)$ at the receiver consists of the three components:

$$S_1(t) = A_1 s(t - \tau_1) \cos \omega(t - \tau_1)$$

$$S_2(t) = A_2 s(t - \tau_2) \cos \omega(t - \tau_2)$$

$$S_3(t) = A_3 s(t - \tau_3) \cos \omega(t - \tau_3)$$

With $\tau_2 = \tau_1 + \frac{1}{2f}$, and f = carrier frequency, $S_1(t)$ and $S_2(t)$ are the components of the direct signal, and $S_3(t)$ is the delayed component. Because the pulse duration T of the modulation is large compared to $\frac{1}{2f}$, $S(t - \tau_1)$ is equal to $S(t - \tau_2)$, except in the vicinity where $S_1(t)$ or $S_2(t)$ change signs. For the cosine terms of $S_1(t)$ and $S_2(t)$, it is

$$\cos \omega(t - \tau_2) = \cos (\omega t - \omega \tau_1 - \omega/2f) = \cos [\omega(t - \tau_1) - \pi] = -\cos \omega(t - \tau_1).$$

Therefore, $S_1(t)$ and $S_2(t)$ cancel out except in the vicinity where $S_1(t)$, or $S_2(t)$ change signs. The remaining component $S_3(t)$, on which the receiver may lock, will cause a range error that is proportional to $\tau_3 - \tau_1$.

In the detected signal of equation (7), only A_o and τ_o are determined by the receiver. Normally τ_o is adjusted by the receiver such that $\cos \omega(\tau_2 - \tau_o) = 1$.

The detected signal is to be correlated with a reference signal that has the form

$$R(t - \delta) = A_r s(t - \delta) \quad (9)$$

where δ is the phase of the modulation. It should be emphasized that the phase of the reference carrier τ_0 and the phase of the reference modulation δ must be retained as independently variable quantities, if a realistic simulation is to be obtained. The cross-correlation function $C(\delta)$ between the received and the reference signal is to be evaluated.

$$C(\delta) = \lim_{L \rightarrow \infty} \frac{1}{2L} \int_{-L}^L R(t - \delta) S_d(t) dt \quad (10)$$

The value of δ that maximizes it will give the time delay and, consequently, the range.

The signal $s(t)$ is composed of a series of pulses, each having duration T , that make up a coded string of N pulses having duration NT . By introducing the function $\sigma(x)$ given on the interval $(0, N)$ by

$$\sigma(x) = \begin{cases} 1; & 0 \leq x < 1 \\ 0; & 1 \leq x < N \end{cases} \quad (11)$$

a single pulse assumes the following form:

$$\sigma\left(\frac{t}{T}\right) = \begin{cases} 1; & 0 \leq t < T \\ 0; & T \leq t \leq NT \end{cases} \quad (12)$$

with $\frac{t}{T} = x$.

Outside of this fundamental interval $0 \leq x < 1$, the function will be defined as periodic of period N :

$$\sigma(x + kN) = \sigma(x) \quad (13)$$

$$\sigma\left(\frac{t + kNT}{T}\right) = \sigma\left(\frac{t}{T}\right) \quad (14)$$

where k assumes positive and negative integral values. The function $\sigma(x)$ is shown in figure 11.

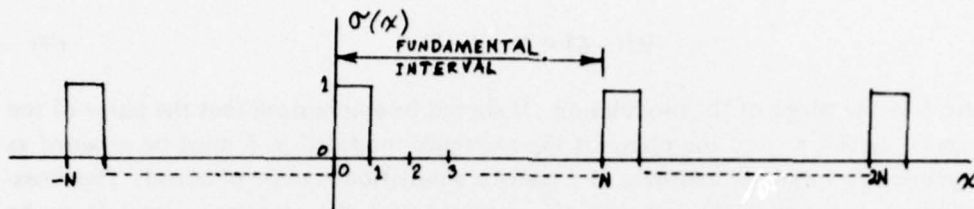


Figure 11. The Function $\sigma(x)$.

The coded string of pulses now assumes the form

$$s(t) = \sum_{j=1}^N a_j \sigma \left(\frac{t - jT}{T} \right) \quad (15)$$

or

$$s(t) = a_j, \quad j < \frac{t}{T} \leq j+1, a_j = \pm 1 \quad (16)$$

the actual values of a_j 's being given by the code. The periodicity of σ accounts for the fact that $s(t)$ is transmitted periodically:

$$s(t + kNT) = s(t), k = 0, \pm 1, \pm 2, \dots \quad (17)$$

The detected signal and the reference signal may now be expressed in more explicit terms in the forms given by equations (18) and (19), respectively.

$$S_d(t) = \sum_{i=1}^M \sum_{j=1}^N B_i a_j \sigma \left(\frac{t - \tau_i - jT}{T} \right) \quad (18)$$

$$R(t - \delta) = \sum_{k=1}^N A_r a_k \sigma \left(\frac{t - \delta - kT}{T} \right) \quad (19)$$

Because of the periodicity of its integrand, the expression for the correlation function given by equation (10) may be replaced by one having an integral over only one period:

$$C(\delta) = \frac{1}{NT} \int_0^{NT} R(t - \delta) S_d(t) dt \quad (20)$$

$$C(\delta) = \frac{A_r}{NT} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k \int_0^{NT} \sigma\left(\frac{t - \tau_i - jT}{T}\right) \sigma\left(\frac{t - \delta - kT}{T}\right) dt \quad (21)$$

By making the substitution

$$t' = t - \delta - kT, \quad (22)$$

equation (21) can be expressed in the following form:

$$C(\delta) = \frac{A_r}{NT} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k \int_{-\delta - kT}^{NT - \delta - kT} \sigma\left(\frac{t' + \delta + kT - \tau_i - jT}{T}\right) \sigma\left(\frac{t'}{T}\right) dt' \quad (23)$$

Again because of periodicity of the integrand, the integral may be taken over any complete period:

$$C(\delta) = \frac{A_r}{NT} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k \int_0^{NT} \sigma\left(\frac{t' + \delta - \tau_i - (j-k)T}{T}\right) \sigma\left(\frac{t'}{T}\right) dt' \quad (24)$$

Using equation (12) to define σ in the second factor of the integrand, the form of the integral may be further simplified:

$$C(\delta) = \frac{A_r}{NT} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k \int_0^T \sigma\left(\frac{t' + \delta - \tau_i - (j-k)T}{T}\right) dt' \quad (25)$$

If ν_i is taken to be the largest integer such that

$$\nu_i \leq \frac{-\delta + \tau_i}{T} \quad (26)$$

and x_i is taken to be the fractional remainder

$$x_i = \frac{-\delta + \tau_i}{T} - \nu_i \quad (27)$$

$$0 \leq x_i < 1 \quad (28)$$

or

$$(\nu_i + x_i) T = -\delta + \tau_i \quad (29)$$

the correlation function assumes the form

$$C(\delta) = \frac{A_r}{NT} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k \int_0^T \sigma \left(\frac{t'}{T} - (\nu_i - j + k) - x_i \right) dt' \quad (30)$$

or letting $t'' = \frac{t'}{T}$

$$C(\delta) = \frac{A_r}{NT} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k T \int_0^1 \sigma (t'' - \nu_i + j - k - x_i) dt'' \quad (31)$$

From the actual physical situation, it is known that signal components for which the inequality

$$0 \leq \tau_i \ll NT \quad (32)$$

does not hold will give only an insignificant contribution to the received signal, since otherwise the signal will have traveled over such a long path (with possible multiple scattering) that it will be severely attenuated. Thus, for correlation, the only realistic phases for the reference signal correspond to

$$0 \leq \delta \ll NT \quad (33)$$

or

$$|\nu_i + x_i| \ll N \quad (34)$$

In evaluating the integral, values of ν_i need only be taken in the following range:

$$|\nu_i| \ll N \quad (35)$$

As an example, more than 500-meter foliage penetration will completely destroy the incident wave; hence, $|\nu_i| < 17$. Within this range of values of ν_i , the integral in the expression for $C(\delta)$ is readily evaluated (figures 12 and 13).

By introducing a periodic analog of the Kronecker δ defined by

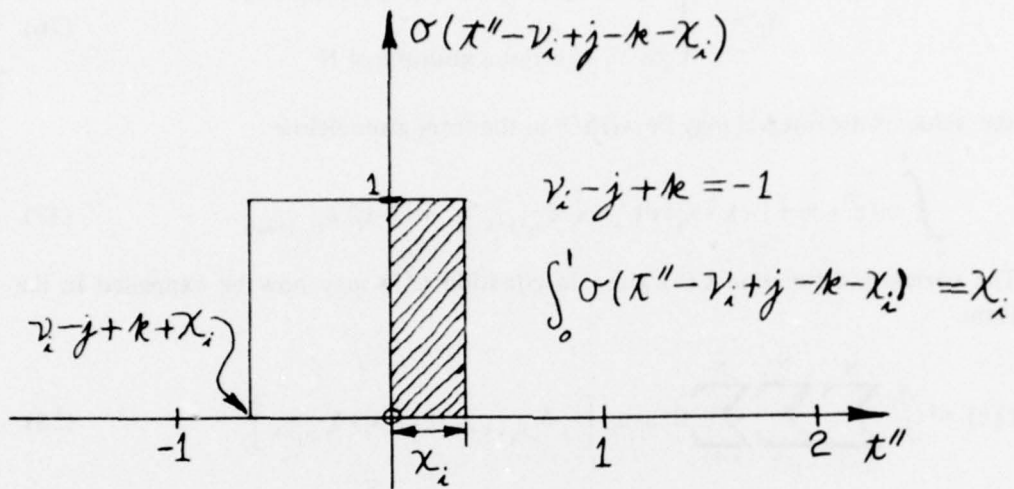


Figure 12. Evaluation of the Integral in $C(\delta)$ for $\nu_i - j + k = -1$.

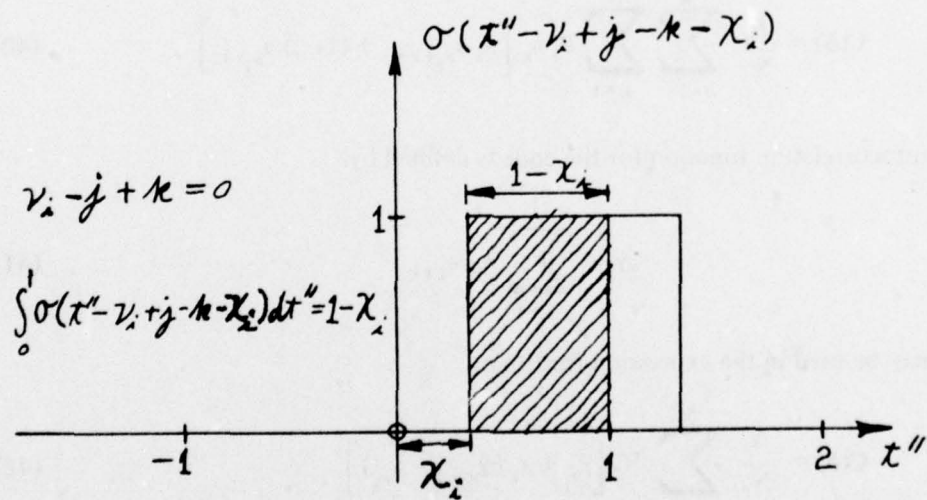


Figure 13. Evaluation of the Integral in $C(\delta)$ for $\nu_i - j + k = 0$.

$$\delta_{ij} = \begin{cases} 1; & i - j = nN, \quad n = 0, \pm 1, \pm 2, \dots \\ 0; & i - j \text{ is not a multiple of } N \end{cases} \quad (36)$$

the value of the integral may be written in the form given below:

$$\int_0^1 \sigma(t'' - \nu_i + j - k - x_i) dt'' = x_i \delta_{\nu_i - j + k, -1} + (1 - x_i) \delta_{\nu_i - j + k, 0} \quad (37)$$

The correlation function $C(\delta)$ given in equation (31) may now be expressed in the form

$$C(\delta) = \frac{A_r}{N} \sum_{i=1}^M \sum_{j=1}^N \sum_{k=1}^N B_i a_j a_k \left[x_i \delta_{j, \nu_i + k + 1} + (1 - x_i) \delta_{j, \nu_i + k} \right] \quad (38)$$

and by periodically extending the range of subscripts of a_j such that

$$a_{j+N} = a_j, \quad (39)$$

summation over j may be performed:

$$C(\delta) = \frac{A_r}{N} \sum_{i=1}^M \sum_{k=1}^N B_i a_k \left[x_i a_{\nu_i + k + 1} + (1 - x_i) a_{\nu_i + k} \right] \quad (40)$$

The autocorrelation function for the code is defined by

$$\gamma_\ell = \sum_{k=1}^N a_k a_{\ell+k} \quad (41)$$

and may be used in the expression for $C(\delta)$:

$$C(\delta) = \frac{A_r}{N} \sum_{i=1}^M B_i \left[\gamma_{\nu_i} + x_i (\gamma_{\nu_i+1} - \gamma_{\nu_i}) \right] \quad (42)$$

If the γ_ℓ 's are calculated once and stored in a computer, they may be used repeatedly in the computation of $C(\delta)$.

Adjustment of the Receiver's Carrier Phase and Equations for Computing the Correlation Function. By using equation (8) and by introducing the coefficients

$$\beta_i = \frac{A_i A_o A_1}{2N} \left[\gamma_{v_i} + x_i (\gamma_{v_i+1} - \gamma_{v_i}) \right] \quad (43)$$

an alternative expression for the correlation function may be written in a form where the carrier phases of the component waves is emphasized:

$$C(\delta) = \sum_{i=1}^M \beta_i \cos \omega (\tau_i - \tau_o) \quad (44)$$

If the carrier phase τ_o of the receiver generated signal is considered to be variable, but all of the other parameters in equation (44) are fixed, then the correlation function $C(\delta)$ may be written as

$$C(\delta) = \tilde{C} \cos \omega (\tilde{\tau} - \tau_o) \quad (45)$$

since a sum of sinusoidal functions of a single frequency is again a sinusoidal function of the same frequency. At the receiver, δ is shifted to find the maximum value of $C(\delta)$, but as δ is shifted τ_o also follows in such a way that $C(\delta)$ is maximized in equation (45) so that $C(\delta) = \tilde{C}$ and $\tau_o = \tilde{\tau} + 2n\pi$ (where $n = \text{integer}$). Again, this is an idealization, and the design and construction of actual receivers may correspond only in this ideal. In actuality, δ is shifted by the voltage controlled oscillator (VCO) that drives the code generator, and τ_o is adjusted by the voltage controlled oscillator (VCO) of the carrier phase lock loop. After signal acquisition, code tracking loop and phase lock loop may operate coherently. From equations (44) and (45),

$$\tilde{C} \cos \omega (\tilde{\tau} - \tau_o) = \sum_{i=1}^M \beta_i \cos \omega (\tau_i - \tau_o) \quad (46)$$

and the functional form of the equation must be preserved for any translation of the τ_o axis:

$$\tilde{C} \sin \omega (\tilde{\tau} - \tau_o) = \sum_{i=1}^M \beta_i \sin \omega (\tau_i - \tau_o) \quad (47)$$

Also, for the same reason, equations (46) and (47) must hold for the particular choice $\tau_o = 0$,

$$\tilde{C} \sin \omega \tilde{\tau} = \sum_{i=1}^M \beta_i \sin \omega \tau_i \quad (48)$$

$$\tilde{C} \cos \omega \tilde{\tau} = \sum_{i=1}^M \beta_i \cos \omega \tau_i \quad (49)$$

and from these relationships, \tilde{C} is readily determined:

$$\tilde{C}^2 = (\tilde{C} \sin \omega \tilde{\tau})^2 + (\tilde{C} \cos \omega \tilde{\tau})^2 \quad (50)$$

$$\tilde{C} = \left[\left(\sum_{i=1}^M \beta_i \sin \omega \tau_i \right)^2 + \left(\sum_{i=1}^M \beta_i \cos \omega \tau_i \right)^2 \right]^{1/2} \quad (51)$$

Equation (51) was used in the calculation of the correlation function \tilde{C} by computer. If desired, the phase $\tilde{\tau}$ can be found from equations (48) and (49) if care is taken in removing phase ambiguities that can arise in calculating inverse trigonometric functions.

The Maximum Value of the Correlation Function. Although equation (51) will suffice for computing $\tilde{C}(\delta)$, computing this function at enough points to insure finding its maximum can still present a formidable problem. However, by introducing a complex function $\bar{C}(\delta)$ and by dividing the δ -axis into certain intervals, an interpretation can be made that will permit $\tilde{C}(\delta)$ to be computed only at a finite and reasonable (although large) number of predetermined points. If $\bar{C}(\delta)$ is given by

$$\bar{C}(\delta) = \sum_{k=1}^M \beta_k e^{i\omega \tau_k} \quad (52)$$

then by using de Moivre's relation for the complex exponentials,

$$\bar{C}(\delta) = \sum_{k=1}^M \beta_k \cos \omega \tau_k + i \sum_{k=1}^M \beta_k \sin \omega \tau_k \quad (53)$$

it is evident that $\tilde{C}(\delta)$ is the modulus or distance from the origin to the point $\bar{C}(\delta)$ in the complex plane:

$$\tilde{C}(\delta) = |\bar{C}(\delta)| = [\bar{C}(\delta) \bar{C}^*(\delta)]^{1/2} \quad (54)$$

If the quantities x'_j are taken to be a permutation of the quantities x_j such that

$$x'_1 \leq x'_2 \leq \dots \leq x'_n, \quad (55)$$

then the particular values of δ denoted by

$$\delta_{m+Mn} = (x'_m + n)T \quad (56)$$

will have the following property.

$$\delta_k \leq \delta_{k+1} \quad (57)$$

Furthermore, examination of equation (43) and (53) reveals that between successive values of δ_k , $\bar{C}(\delta)$ is a linear function of δ and will move in the complex plane along the straight line joining the two points $\bar{C}(\delta_k)$ and $\bar{C}(\delta_{k+1})$ (figure 14),

$$\bar{C}(\delta) = \bar{C}(\delta_k) + \frac{\delta - \delta_k}{\delta_{k+1} - \delta_k} [\bar{C}(\delta_{k+1}) - \bar{C}(\delta_k)]; \delta_k \leq \delta \leq \delta_{k+1} \quad (58)$$

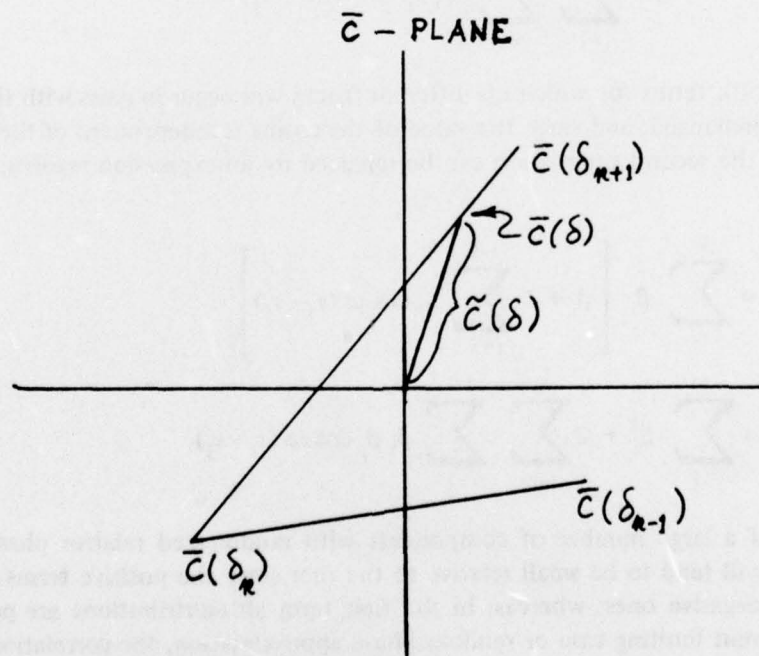


Figure 14. Complex Plane Construction for the Correlation Function.

Within each interval defined in this way, $\tilde{C}(\delta)$ can have at most one minimum value, and its maximum value must occur at one of the endpoints. It can easily be shown that on any one of the intervals, $\tilde{C}(\delta)$ is represented by a conic section. The endpoints of the intervals are the points that have been calculated in the computer program. Thus, for a code of length N and for M components, the correlation function $\tilde{C}(\delta)$ needs to be computed only at NM points to insure finding its maximum.

The Incoherent Limiting Case. In the case where a very large number of component waves are present and all have about equal amplitude and randomized phases, the correlation function can be shown to be independent of the carrier phase. Equation (51) may be rewritten in the form

$$\tilde{C}^2 = \sum_{i=1}^M \sum_{j=1}^M \beta_i \beta_j (\cos \omega \tau_i \cos \omega \tau_j + \sin \omega \tau_i \sin \omega \tau_j) \quad (59)$$

or, by using a familiar trigonometric identity, in the following form:

$$\tilde{C}^2 = \sum_{i=1}^M \sum_{j=1}^M \beta_i \beta_j \cos \omega (\tau_i - \tau_j) \quad (60)$$

In equation (60), terms for which i is different from j will occur in pairs with the roles of i and j interchanged; and since the value of the cosine is independent of the sign of its argument, the second summation can be replaced by an expression involving fewer terms:

$$\tilde{C}^2 = \sum_{i=1}^M \beta_i \left[\beta_i + 2 \sum_{j=1}^{i-1} \beta_j \cos \omega (\tau_i - \tau_j) \right] \quad (61)$$

$$\tilde{C}^2 = \sum_{i=1}^M \beta_i^2 + 2 \sum_{i=1}^M \sum_{j=1}^{i-1} \beta_i \beta_j \cos \omega (\tau_i - \tau_j) \quad (62)$$

In the case of a large number of components with randomized relative phases, the second term will tend to be small relative to the first since the positive terms can be cancelled by negative ones; whereas, in the first term all contributions are positive. In this incoherent limiting case or random phase approximation, the correlation function becomes independent of the carrier phases of the components

$$\hat{C}(\delta) = \left[\sum_{i=1}^M \beta_i^2 \right]^{1/2} \quad (63)$$

and this corresponds with the presence of many multipath signals of about equal amplitude whose path lengths are randomized relative to a wavelength.

DISCUSSION OF COMPUTED EXAMPLES

By using the formulas and methods previously outlined, a computer program was written and the results were computed for examples that seemed to conform reasonably with the experimental situation and that would demonstrate the phenomenon of carrier cancellation. Examples of correlation functions are given in figures 15 and 16. Both examples show results for seven components with the assigned amplitudes and path lengths shown below:

Range (meters)	Amplitude
200.0 m	1.00
201.0 + δ m	0.95
230.07442 m	0.20
260.05367 m	0.20
290.03292 m	0.20
320.01217 m	0.20
349.99142 m	0.20

For the correlation function in figure 15, the path increment δ corresponded to 270° in carrier wavelength, and in figure 16, δ corresponded to 90° .

The data are plotted again in figures 17 and 18 for comparison purposes. Here, suggestive connecting curves have been added, although their exact form is not known. The change at around 200 meters is a clear example of carrier cancellation. In this example, either one of the first two components contain more than eight times the combined energy of the remaining five components; yet, in the case of carrier cancellation, the latter five components all show correlation values at least twice as high as that of the two larger components. This all occurs because the path length of one of the strong components is changed by half a wavelength, or about 10 centimeters. It should be noted that the five minor components shown appear to be almost completely unaltered by the large changes associated with the two principal components. The reason that they remained unaltered is that the minor components are separated from the principal components in path length by a pulse length or more.

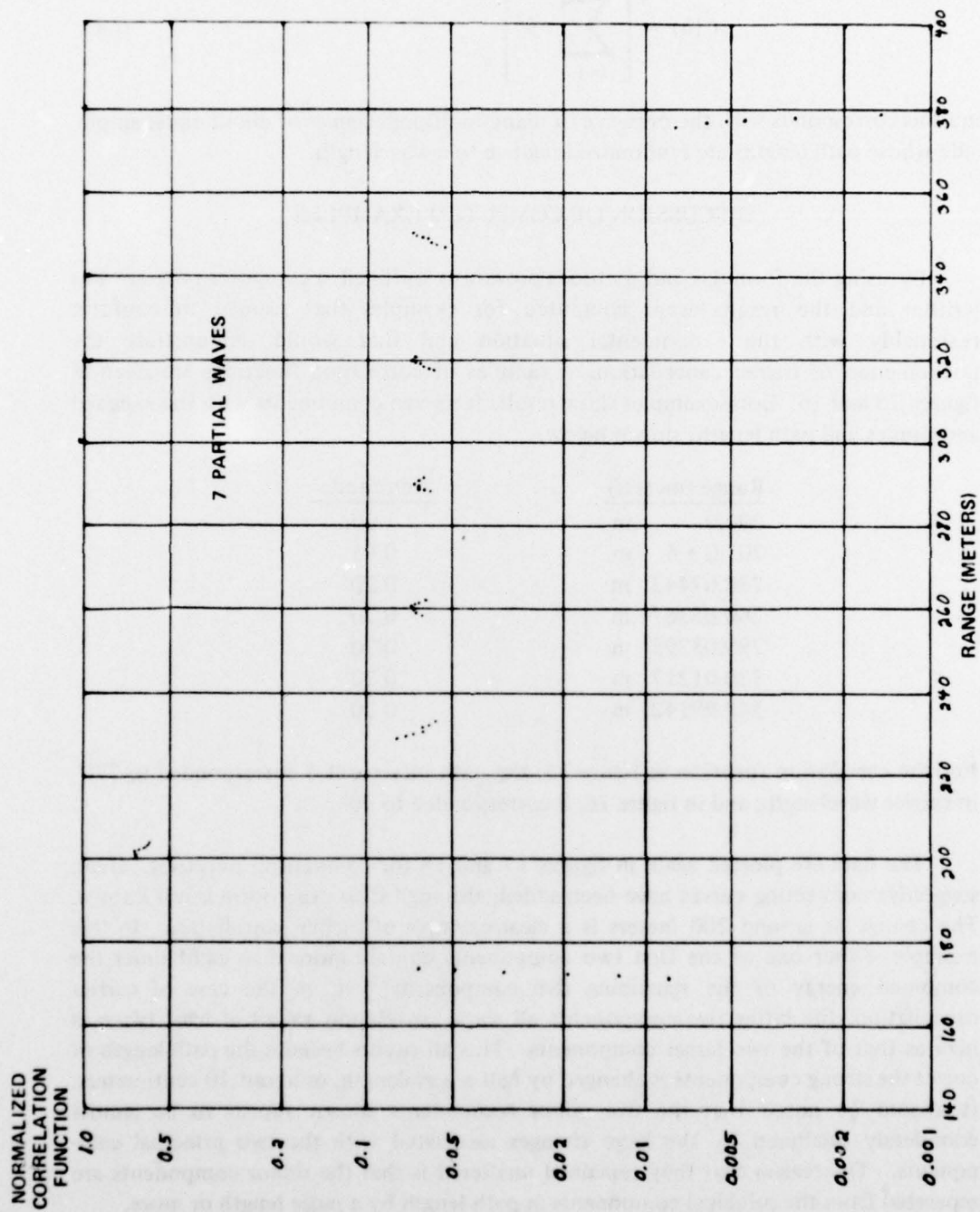


Figure 15. First Correlation Function.

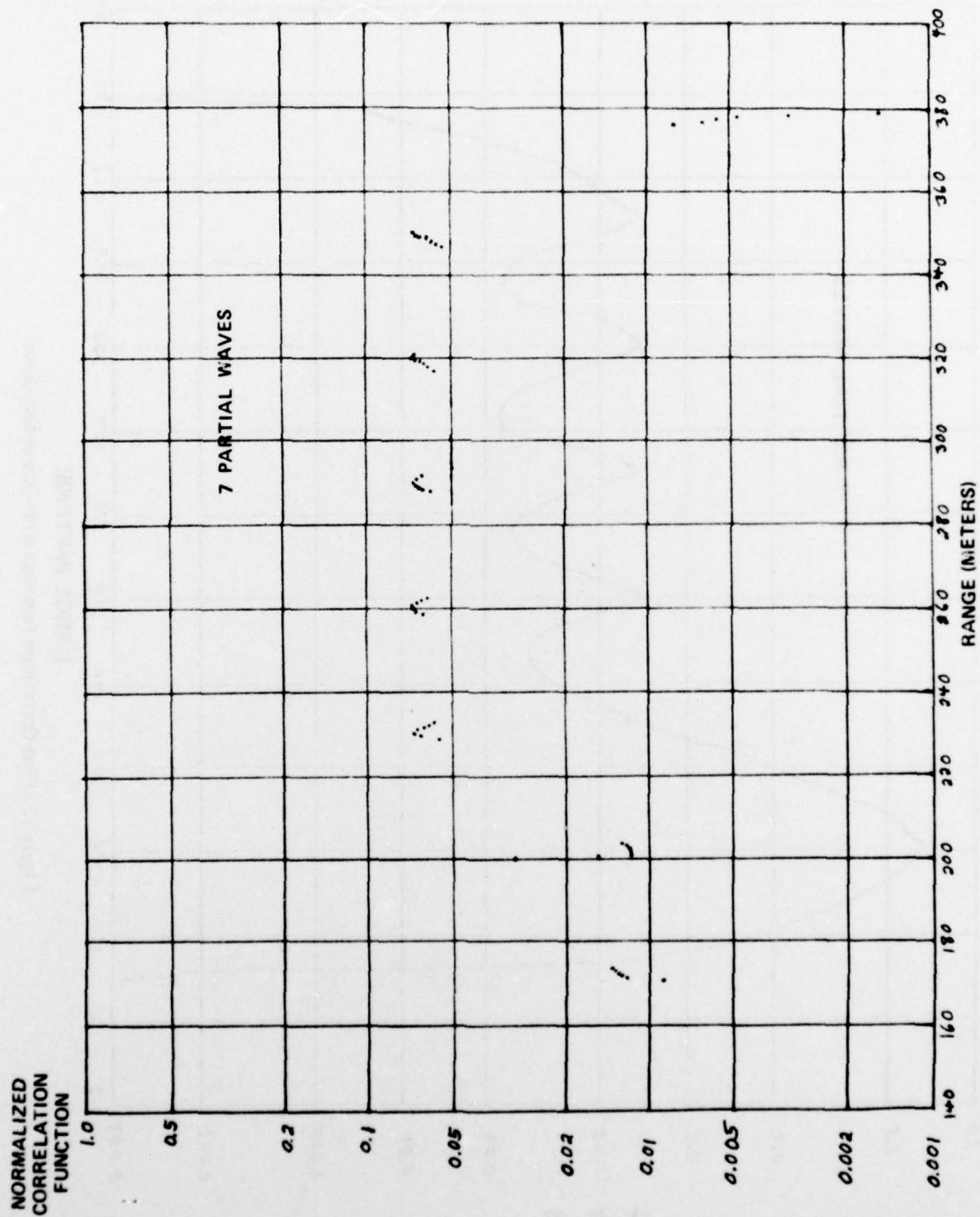


Figure 16. Second Correlation Function.

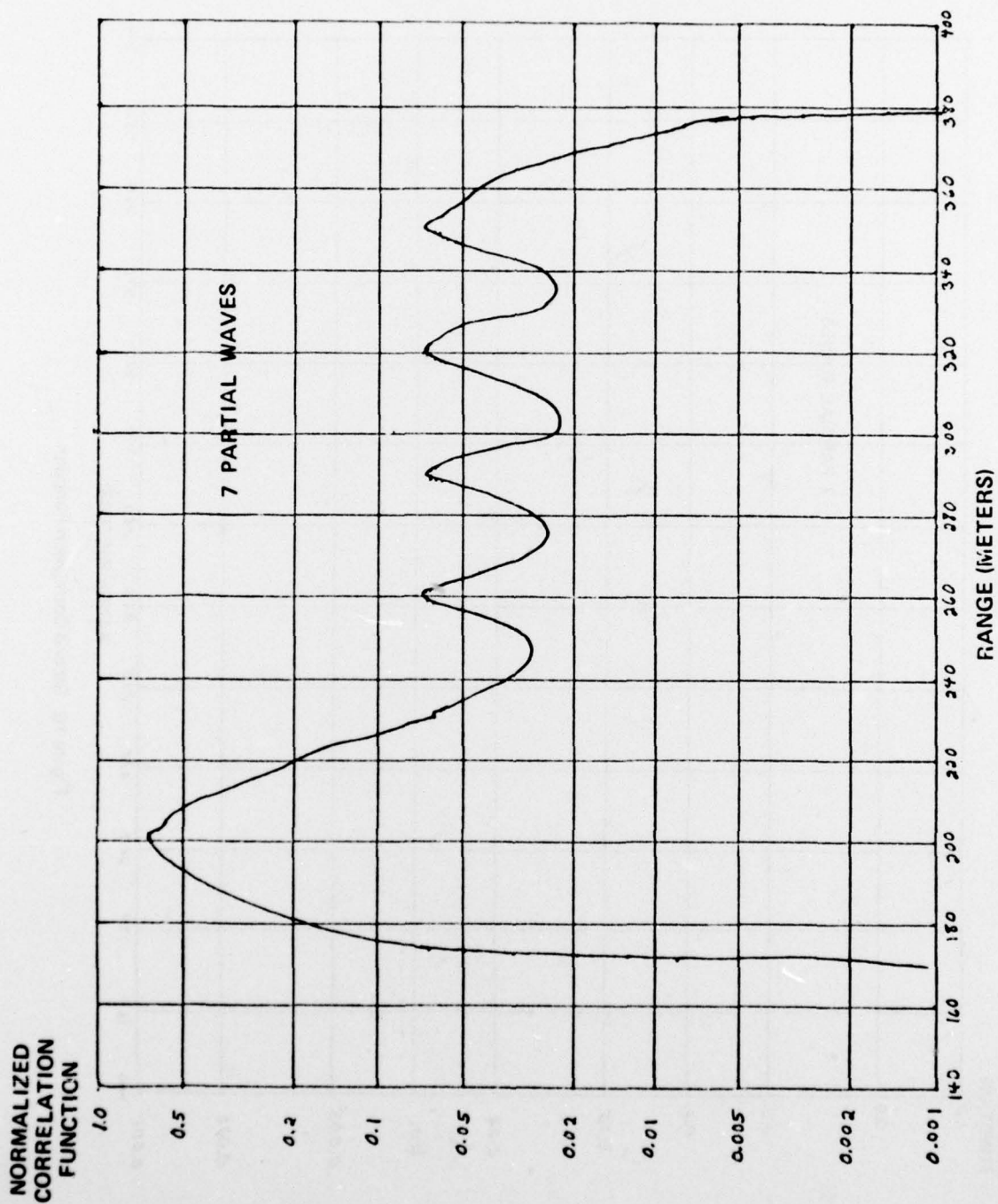


Figure 17. First Correlation Function with Connecting Curve.

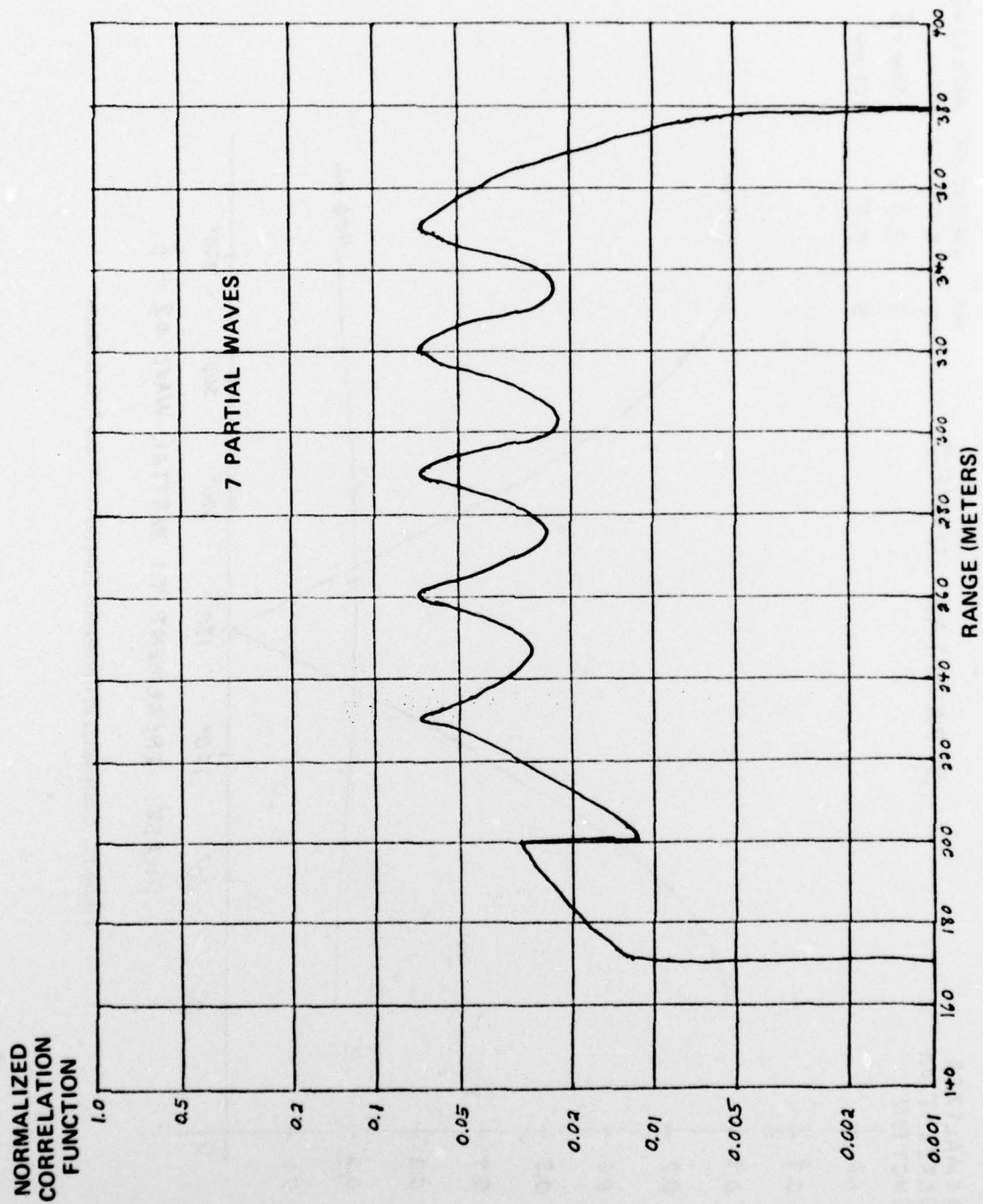


Figure 18. Second Correlation Function with Connecting Curve.

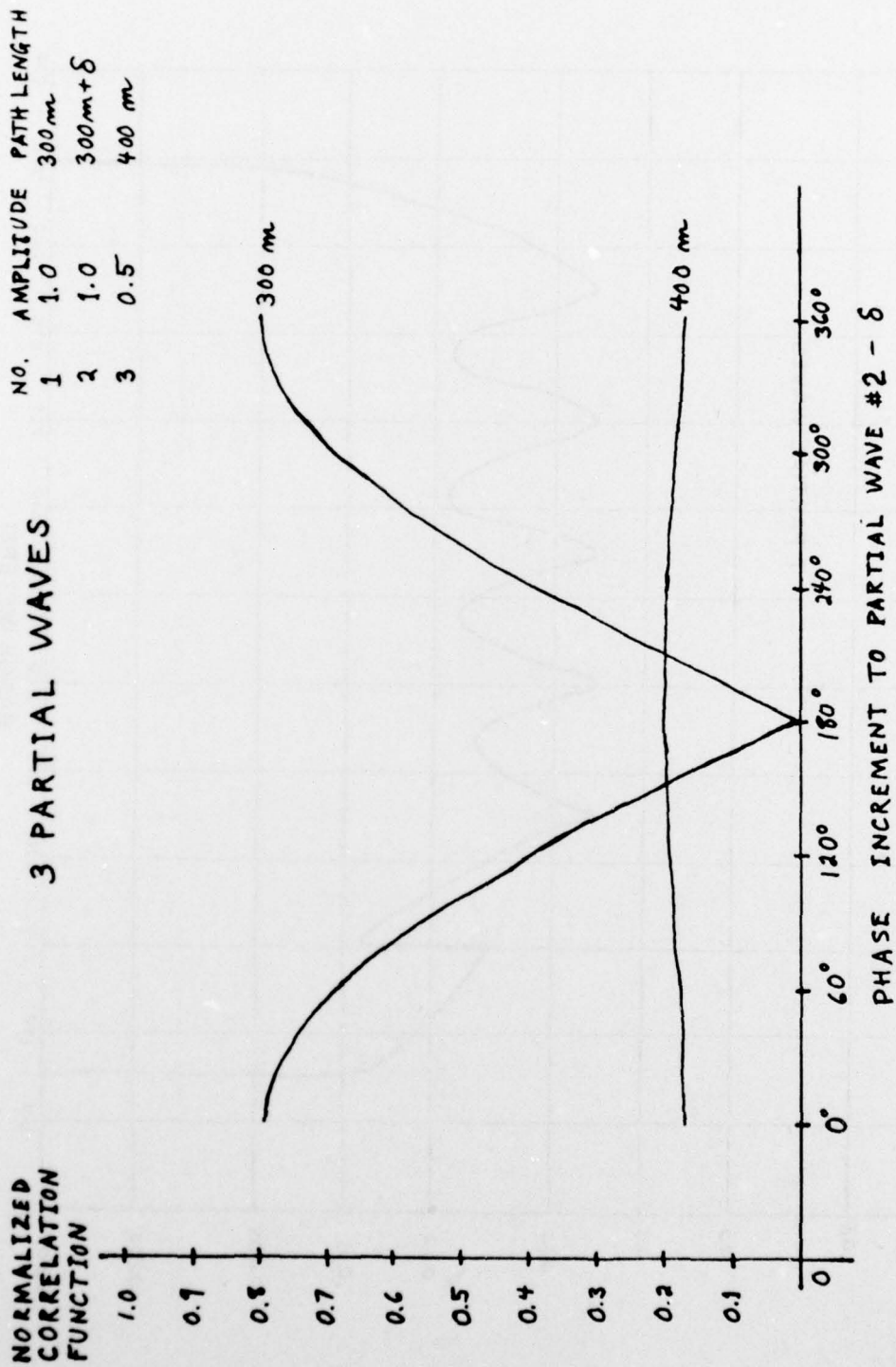


Figure 19. Dependence of Correlation Function on Phase Relationships.

The change in correlation peaks can be followed as the phase of one of the components is changed. For example, the curves in figure 19 are plotted for three component waves corresponding with the following ranges and amplitudes:

<u>Range (meters)</u>		<u>Amplitude</u>
300.0	m	1.0
$300.0 + \delta$	m	1.0
400.0	m	0.5

In this case, the abscissa gives the increase in the path length δ associated with component 2. The correlation peaks that are followed correspond to 300 and 400 meters, respectively. The complete cancellation in the 300-meter correlation peak occurs because the first two components have exactly the same amplitude and almost no range separation.

IV. CONCLUSIONS

It is concluded that

1. The proposed mechanism of carrier cancellation is a plausible explanation for anomalous range measurements obtained in a scattering environment using a code modulated signal.
2. Carrier cancellation is a property of the scattered signal only and cannot be influenced by receiver signal processing.
3. Carrier cancellation may occur when the path difference of the interfering signals is less than the length of the modulation pulse.
4. In the incoherent limiting case where a large number of component waves are present, the correlation function becomes independent of the carrier phases of the components. Then, a receiver will lock onto the signal only when the correlation function has distinctive peaks.
5. The probability of large ranging errors due to carrier cancellation is inversely related to the modulation bit rate.

APPENDIX

THE COMPUTER PROGRAM

The computations were performed on a CDC 6600 computer using a program written in FORTRAN. The program consisted of a main program and three subroutines. Subroutine CODEIN is used to generate the Gold Code and to store it in a COMMON area. Its argument, NADVA, specifies the relative advance of the two shift registers. The amount of relative advance may be changed without altering the configuration of shift register connections by using ENTRY SHFTADV. The autocorrelation function for the code is computed by calling subroutine CORVAL, which in turn computes each value of the autocorrelation function by calling subroutine CODECOR. The main program computes the correlation function of the multicomponent signal with the receiver generated signal. This is computed at all of the predetermined points (mentioned previously). The correlation function is printed out in detail for the principal range (or range differences) of interest. For the rest of the correlation function, the largest values and their corresponding ranges are stored, and only these are printed out. A list of input data with the corresponding card formats is given below followed by a listing of the actual computer programs.

INPUT DATA

			<u>Field Specification</u>	<u>Card Column</u>
CARD 1	NCC	— Number of Computed Simulations to be Run	I5	1-5
	FREQ	— Frequency of Carrier (Hertz)	E15	6-20
	PULF	— Pulse Frequency (1.023 E+07 (bit/sec))	E15	21-35
	VEL	— Velocity of Propagation (Meters/Second)	E15	36-50
CARD 2	Contains shift register configuration for the two shift registers used in generating the code, e.g. 0010000001, 0110010111 specifies connections for feedback of the sum (mod 2) from the 3d and 10th registers of the first shift register and from the 2d, 3d, 6th, 8th, 9th, and 10th register in the second.		1011	1-10
			1011	21-30

NCC Card Sets for each Complete Simulation

CARD 1	NPW	— Number of Partial Waves	I5	1-5
	NCRV	— Number of Correlation Values to be Computed	I5	11-15
	IADVA	— Relative Advance of the first Shift register with respect to the second	I5	21-25

NPW Card Sets for each Partial Wave (One Card per Set)

CARD 1	PL	— Path Length for the Partial Wave (meters)	E15	1-15
	AMP	— Amplitude of the Partial Wave (arbitrary units)	F5	21-25
	NPI	— Number of Phase Increments	I5	31-35
	TPI	— Total Phase Increment or Range of Phase Angles to be Covered (degrees)	F5	41-45

```

1      PROGRAM CODCOR(INPUT,OUTPUT,TAPES=INPUT,TAPE6=OUTPUT)
      COMMON IC(1023),ICRVL(1023),IA(10),IB(10)
      DIMENSION PL(10),AMP(10),NPI(10),TPI(10),CHI(10),CHIO(10),IGAM(10)
      *,BESV(10),BECN(10),DIST(15),CORP(15),CORR(15),CORI(15)
5      PI=3.14159265358979
      READ 10,NCC,FREQ,PULF,VEL
10     FORMAT(I5,3E15.0)
      WVL=VEL/FREQ
      PUL=VEL/PULF
      AFAC=WVL/360.0
      QFAC=2.0*PI/WVL
      REF=2.0
      IADVA=0
      CALL CODEIN(IADVA)
15     CALL CORVAL
      DO 400 NNN=1,NCC
      PRINT 20,FREQ,PULF,VEL,NCC
20     FORMAT(26H1CARRIER FREQUENCY (HZ) = ,1P1E15.6,/,
      *29H PULSE FREQUENCY (BIT/SEC) = ,1P1E15.6,/,25
20     *H VELOCITY (METERS/SEC) = ,1P1E15.6,/,25H NUMBER OF SIMULATIONS =
      *,I5,/)
      PRINT 30,(IA(J),J=1,10),(IB(J),J=1,10)
30     FORMAT(32H SHIFT REGISTER CONFIGURATION - ,10I1,2X,10I1)
      IBOVA=IADVA
25     READ 40,NPW,NCRV,IADVA
40     FORMAT(1I5,5X,1I5,5X,1I5)
      NADVA=IADVA-IBOVA
      IF(NADVA.EQ.0) GO TO 50
      CALL SHFTADV(NADVA)
      CALL CORVAL
30     PRINT 60,IADVA,(IC(J),J=1,1023)
60     FORMAT(35H RELATIVE SHIFT REGISTER ADVANCE = ,1I4,/,10H GOLD CODE
      *,/,4I(1H ,25I5,/)
      PRINT 70,(ICRVL(J),J=1,1023)
35     FORMAT(31H GOLD CODE CORRELATION FUNCTION,/,8F(1H ,12I10,/)
      READ 110,(PL(JJ),AMP(JJ),NPI(JJ),TPI(JJ),JJ=1,NPW)
110    FORMAT(1E15.5,5X,1F5.3,5X,1I5,5X,1F5.3)
      PRINT 120,NNN,NPW
120    FORMAT(////,21H SIMULATION NUMBER = ,1I5,10X,26H NUMBER OF PARTIAL
40    * WAVES = ,1I2,////,7H NUMBER,8X,11H PATH LENGTH,9X,3H AMPLITUDE,8X,
      *10H NO OF INCS,5X,13H TOT INC (DEG),/)
      PRINT 130,(JJ,PL(JJ),AMP(JJ),NPI(JJ),TPI(JJ),JJ=1,NPW)
130    FORMAT(1X,1I5,6X,1P1E15.6,5X,1P1E15.6,8X,1I5,10X,0P1F9.3,/)
      ANORM=0.0
45     DO 150 II=1,NPW
      IGAM(II)=PL(II)/PUL
      CHI(II)=PL(II)-FLOAT(IGAM(II))*PUL
      CHIO(II)=CHI(II)
      ANORM=ANORM+AMP(II)
50     150 CONTINUE
      XNORM=REF/(ANORM*2046.0)
      DO 400 II=1,NPW
      IF(II.EQ.1) GO TO 160
      IF(NPI(II).EQ.0) GO TO 400
55     160 PLTE4=PL(II)
      CHITE4=CHI(II)
      IGTE4=IGAM(II)
      IIT=NPI(II)+1
      IF(NPI(II).EQ.0) GO TO 170
      DELANG=TPI(II)*AFAC/FLOAT(NPI(II))
      ANGI=TPI(II)/FLOAT(NPI(II))
      GO TO 180
60     170 DELANG=0.0
      ANGI=0.0
      180 DO 300 JJ=1,IIT

```



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      DJJ=JJ-1
      PL(II)=PLTEN+DJJ*DELANG
      IGAM(II)=PL(II)/PULL
      CHI(II)=PL(II)-FLOAT(IGAM(II))*PULL
70      DO 190 KK=1,NPW
      CHIO(KK)=CHI(KK)
190     CONTINUE
      ANG=DJJ*ANGI
      NPWM=NPW-1
75      DO 200 KK=1,NPWH
      KKK=KK+1
      DO 200 LL=KKK,NPW
      IF(CHIO(LL).GE.CHIO(KK)) GO TO 200
      CHIB=CHIO(LL)
80      CHIO(LL)=CHIO(KK)
      CHIO(KK)=CHIB
200     CONTINUE
      DO 210 KK=1,NPW
      ARG=QFAC*PL(KK)
85      BESN(KK)=SIN(ARG)
      BECN(KK)=COS(ARG)
210     CONTINUE
      PRINT 220,II,ANG
220     FORMAT(23H1PARTIAL WAVE BEING VARIED = ,I2,/,
*37H ANGULAR PHASE INCREMENT (DEGREES) = ,I6.2,/,
*44H CORRELATION FUNCTION (IN RANGE OF INTEREST),/)
90      PRINT 230
230     FORMAT(2X,3(43H RANGE      AMPLITUDE REAL PART IMAG PART      ),/)
      NN=1
95      NCRVA=0
      DO 290 KK=1,15
      DO 290 LL=1,NPW
      NCRVA=NCRVA+1
      CORR(NN)=0.0
100     CORI(NN)=0.0
      DELT=FLOAT(KK-1)*CHIO(LL)/PULL
      DO 250 MM=1,NPW
      RNG=>L(MM)/PULL-DELT
      IF(RNG.LT.1.0)RNG=RNG+1023.0
105     NUMH=RNG
      CHIMH=RNG-FLOAT(NUMH)
      NUMP=NUMH+1
      IF(NUMP.GT.1023) NUMP=NUMP-1023
      BETB=AMP(MM)*(ICRVL(NUMH)+CHIMH*(ICRVL(NUMH)-ICRVL(NUMH)))
110     CORR(NN)=CORR(NN)+BETB*BECN(MM)
      CORI(NN)=CORI(NN)+BETB*BESN(MM)
250     CONTINUE
      CURR(NN)=XNORM*CORR(NN)
      CORI(NN)=XNORM*CORI(NN)
115     DIST(NN)=PULL*DELT
      CORP(NN)=SQRT(CORR(NN)*CURR(NN)+CORI(NN)*CORI(NN))
      IF(NN.LT.3) GO TO 290
      PRINT 260,(DIST(NH),CORP(NH),CORR(NH),CORI(NH),NH=1,3)
120     FORMAT(14,3(1P4E10.2,3X))
      NN=0
280     NN=NN+1
      IF(NCRVA-NCRV.GE.3)GO TO 295
290     CONTINUE
      IF(NCRVA.GE.NCRV) GO TO 380
125     PRINT 300
300     FORMAT(//,53H CORRELATION FUNCTION (OUTSIDE PRINCIPLE RANGE OF IN
*TEREST),/)
      PRINT 230
      DO 310 NN=1,15
      CORP(NN)=0.0
130

```

```

CORR(NN)=0.0
CORI(NN)=0.0
310 CONTINUE
DO 350 KK=16,1023
135 DO 350 LL=1,NPW
NCRVA=NCRVA+1
CORRA=0.0
CORIA=0.0
DELT=FLOAT(KK-1)*CHI(11)/PULL
140 DO 320 MM=1,NPW
RNG=PL(MM)/PULL-DELT
NUMM=RNG
CHIMM=RNG-FLOAT(NUMM)
IF(NUMM.LT.1) NUMM=NUMM+1023
NUMP=NUMM+1
145 IF(NUMP.GT.1023) NUMP=NUMP-1023
DETB=AMP(MM)*(ICRVL(NUMM)+CHIMM*(ICPVL(NUMP)-ICRVL(NUMM)))
CORRA=CORRA+DETB*BEON(MM)
CORIA=CORIA+DETB*BEON(MM)
150 320 CONTINUE
CORRA=XNUMM*CORRA
CORIA=XNUMM*CORIA
DISTA=PULL*DELT
CORPA=SQRT(CORRA*CORRA+CORIA*CORIA)
155 DO 345 MM=1,15
IF(CORPA.LE.CORP(MM)) GO TO 345
IF(MM.GE.15) GO TO 340
NM=15-MM
160 DO 330 NN=1,4M
NA=15-NN
NP=15-NN
CORP(NN)=CORP(NA)
CORR(NN)=CORR(NA)
CORI(NN)=CORI(NA)
DIST(NN)=DIST(NA)
165 330 CONTINUE
340 CORP(MM)=CORP1
CORI(MM)=CORIA
CORP(MM)=CORP2
DIST(MM)=DISTA
170 IF(NCRVA-NCRV.GE.0) GO TO 360
GO TO 350
345 CONTINUE
350 CONTINUE
175 360 NNAA=LL+(KK-16)*NPW
IF(NNAA.GE.15) NNAA=15
PRINT 370,(DIST(NN),CORP(NN),CORR(NN),CORI(NN),NN=1,NNAA)
370 FORMAT(3(1X,1P+E10.2,2X))
380 CONTINUE
180 PL(11)=PLTEM
CHI(11)=CHITEM
IGAM(11)=IGTEM
400 CONTINUE
STOP
185 END

1 SUBROUTINE CORVAL
COMMON IC(1023),ICRVL(1023),IA(10),IB(10)
DO 100 KDIF=1,1023
5 CALL CODECOR(KDIF,ICRVL(KDIF))
100 CONTINUE
RETURN
END

```

```

1      SUBROUTINE CODFIN(NADVA)
      DIMENSION ISRA(10),ISRB(10)
      COMMON IC(1023),ICRVL(1023),IA(10),IB(10)
      READ 100,(IA(J),J=1,10),(IB(J),J=1,10)
5      100 FORMAT(10I1,10X,10I1)
      DO 150 J=1,10
      ISRA(J)=1
      ISRB(J)=1
10     150 CONTINUE
      ENTRY SHFTADV
200    IF(NADVA.GT.0) GO TO 210
      IF(NADVA.LE.0)GO TO 250
      NADVA=1023+NADVA
      GO TO 200
15     210 DO 240 J=1,NADVA
      NA=0
      DO 220 K=1,10
      NA=NA+IA(K)*ISRA(K)
20     220 CONTINUE
      DO 230 K=1,9
      L=11-K
      LL=10-K
      ISRA(L)=ISRA(LL)
25     230 CONTINUE
      ISRA(1)=NA-2*(NA/2)
240    CONTINUE
250    DO 500 J=1,1023
      NA=0
      NB=0
30     DO 300 K=1,10
      NA=NA+IA(K)*ISRA(K)
      NB=NB+IB(K)*ISRB(K)
300    CONTINUE
      IC(J)=-1
35     IF(ISRA(10).EQ.ISRB(10))IC(J)=1
      DO 400 K=1,9
      L=11-K
      LL=10-K
      ISRA(L)=ISRA(LL)
40     ISRB(L)=ISRB(LL)
400    CONTINUE
      ISRA(1)=NA-2*(NA/2)
      ISRB(1)=NB-2*(NB/2)
45     500 CONTINUE
      RETURN
      END

```

```

1      SUBROUTINE CODECOR(KDAF,ISUM)
      COMMON IC(1023),ICRVL(1023),IA(10),IB(10)
      KDIF=KDAF
10     IF(KDIF.GE.0) GO TO 20
      KDIF=KDIF+1023
5      GO TO 10
20     IF(KDIF.GE.1023)KDIF=KDIF-1023*(KDIF/1023)
      ISUM=0
      DO 100 I=1,1023
10     K=I+KDIF
      IF(K.GT.1023)K=K-1023
      ISUM=ISUM+IC(I)*IC(K)
100    CONTINUE
      RETURN
15     END

```

Margerum, Eugene A.

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